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2 + 1 flavor simulations of QCD with improved staggered quarks

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Abstract. The MILC Collaboration has been performing realistic simulations of full QCD with 2+1 flavors of improved staggered quarks. Our simulations allow for controlled continuum and chiral extrapolations. I present results for the light pseudoscalar sector: masses and decay constants, quark masses and Gasser-Leutwyler low-energy constants. In addition I will present some results for heavy-light mesons, decay constants and semileptonic form factors, obtained in collaboration with the HPQCD and Fermilab lattice Collaborations. Such calculations will help in the extraction of CKM matrix elements from experimental measurements.

PACS. 12.38.Gc Lattice QCD calculations – 12.39.Fe Chiral Lagrangians – 12.15.Hh Determination of Kobayashi-Maskawa matrix elements – 13.20.Fc Decays of charmed mesons

1 Introduction

QCD simulations with the effects of three light quark flavors fully included have been a long-standing goal of lattice gauge theorists. The MILC Collaboration has made significant advances towards this goal, employing an improved staggered formalism, "asqtad" fermions [1]. This formalism, and the ever-increasing computational resources available today, have enabled simulations with the strange-quark mass near its physical value, and the up and down quarks, taken to be of equal mass, as light as 1/10 the strange-quark mass, corresponding to pion masses as low as 240 MeV. To control the extrapolation to the continuum limit, simulations have been done, and are still ongoing, at multiple lattice spacings, $a \approx 0.15$, 0.12, 0.09 and 0.06 fm. These are referred to as coarser, coarse, fine and super-fine lattices, respectively.

All the lattices generated by the MILC Collaboration are made available to other researchers at the "NERSC Gauge Connection". They have been widely used, for example by the LHPC and NPLQCD Collaborations for the study of nucleon properties, form factors and scattering lengths, and by the Fermilab lattice, HPQCD and UKQCD Collaborations for the study of heavy quarkonia and heavy-light mesons. The last three joined efforts with the MILC Collaboration to validate these QCD simulations. They compared selected, "gold-plated", *i.e.* well controlled, quantities computed on the lattice with their experimentally well-known values. Agreement within errors of 1–3% was found [2].

Such a validation is, of course, valuable for any lattice simulation. It is, in particular, needed for simulations with staggered fermions, which rely on the so-called "fourth-root trick" to eliminate the extra species of fermions, called "tastes", present with staggered fermions, and thus to generate the correct number of sea quark flavors. A nice review of the validity or possible problems with this fourth-root trick can be found in [3].

While the MILC Collaborations has made simulations with lighter up and down quark masses, m_l, than have been reached by other groups, simulations at the physical light quark mass are still too costly, even with the improved "asqtad" quarks. Therefore, extrapolations in the light-quark mass —chiral extrapolations— are still needed to reach the physical value. Such extrapolations are well understood, theoretically, based on chiral perturbation theory, χPT. For results from lattice QCD, it is important to include a treatment of lattice effects into the chiral extrapolations. This holds, in particular, for simulation results with staggered fermions, because of the taste symmetry-breaking effects. For this purpose, χPT was adapted to staggered quarks in staggered chiral perturbation theory, SχPT [4].

MILC's first set of accurate results for light pseudoscalars, based on two lattice spacings (coarse and fine, *i.e.* 0.12 and 0.09 fm), one simulation strange quark mass m'_s and several simulation light-quark masses m'_l[1], with the SχPT formalism for joint chiral and continuum extrapolations, was published in 2004 [5]. To obtain quark masses, mass renormalization constants are needed. The

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[1] We denote simulation masses by a prime, m'.

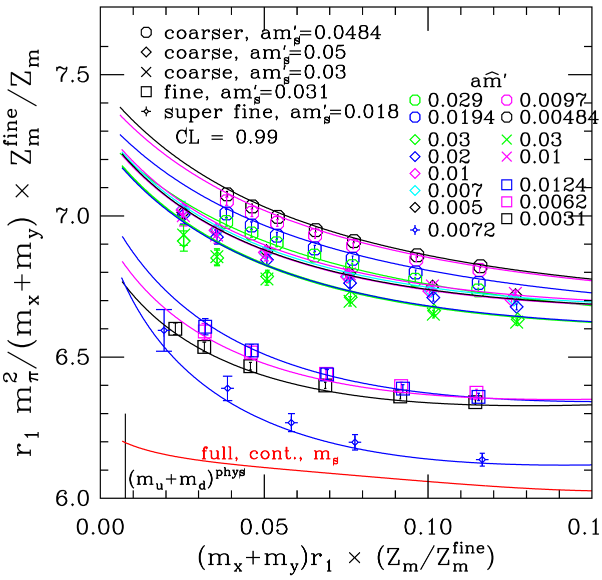

Fig. 1. (Color on-line) The square of the pion mass divided by the sum of the valence quark masses as a function of the sum of the valence quark masses in units of r_1.

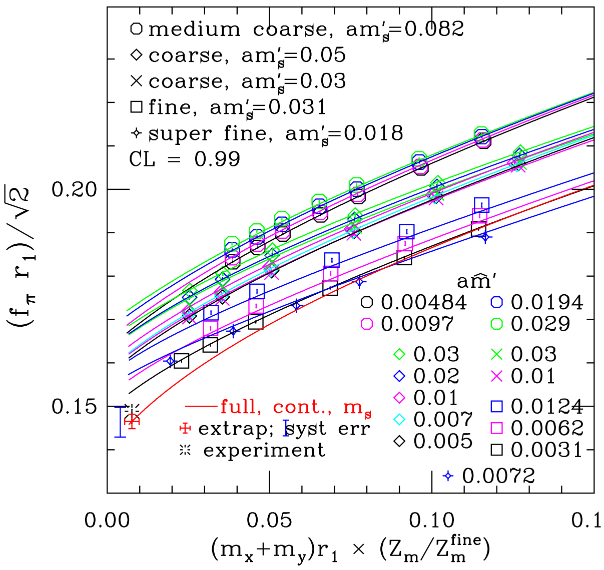

Fig. 2. (Color on-line) The pion decay constant as a function of the sum of the valence quark masses in units of r_1.

values with one-loop Z-factors, computed by members of the HPQCD and UKQCD Collaborations, appeared in [6].

Since 2004, our simulations expanded in several ways. On the coarse, $a = 0.12$ fm, lattice a second simulation strange-quark mass m_s' has been used, allowing interpolation to the physical strange-quark mass. On the fine, $a = 0.09$ fm, lattice a simulation with a lighter light-quark mass, $m_l' \simeq 0.1 m_s$, has been done. A coarser lattice ensemble, with $a = 0.15$ fm, has been added, increasing the lever arm for continuum extrapolations. And, finally, a super-fine-lattice ensemble, with $a = 0.06$ fm, has been started. A run with $m_l' = 0.4 m_s'$, corresponding to $m_\pi \approx 430$ MeV is half-finished, and lighter-mass simulations have begun.

2 The light pseudoscalar sector

The SχPT fitting is illustrated in figs. 1 and 2. Dimensionful quantities are given in units of $r_1 = 0.318(7)$ fm, a scale related to the heavy-quark potential [7]. The quark masses are renormalized (at one loop) relative to those on the fine lattice. The data is accurate enough and at small enough quark masses so that the effects of chiral logs are evident. The red on-line lines are the fit functions in "full continuum QCD" (valence and sea quark masses set equal) after extrapolation of parameters to the continuum limit.

Preliminary numerical results for f_π and f_K obtained from our most recent data are[2]

$$f_\pi = 128.6 \pm 0.4 \pm 3.0\,\text{MeV} \quad [129.5 \pm 0.9 \pm 3.5\,\text{MeV}], \quad (1)$$

$$f_K = 155.3 \pm 0.4 \pm 3.1\,\text{MeV} \quad [156.6 \pm 1.0 \pm 3.6\,\text{MeV}], \quad (2)$$

$$f_K/f_\pi = 1.208(2)(^{+\,7}_{-14}) \quad [1.210(4)(13)]. \quad (3)$$

Here the numbers on the left are the new values, and those on the right in square brackets are from ref. [5]. In each case the first error is statistical, and the second systematic.

The lattice QCD value of f_K/f_π can be combined with experimental data to extract the CKM matrix element V_{us} [9]. We obtain $|V_{us}| = 0.2223(^{+26}_{-14})$, with an accuracy comparable to, and a value compatible with, the latest (2006) PDG value, $V_{us} = 0.2257(21)$ [10].

The up-, down- and strange-quark masses can be determined from the masses of the π and K mesons using the SχPT fits. To relate these masses to the experimental ones requires some continuum input for isospin breaking and electromagnetic effects [11]. Details can be found in ref. [5]. Preliminary results from our current data set, evaluated at scale $\mu = 2$ GeV, are

$$m_s^{\overline{\text{MS}}} = 90(0)(5)(4)(0)\,\text{MeV} \quad [76(0)(3)(7)(0)\,\text{MeV}], \quad (4)$$

$$m_u^{\overline{\text{MS}}} = 2.0(0)(1)(1)(1)\,\text{MeV} \quad [1.7(0)(1)(2)(2)\,\text{MeV}], \quad (5)$$

$$m_d^{\overline{\text{MS}}} = 4.6(0)(2)(2)(1)\,\text{MeV} \quad [3.9(0)(1)(4)(2)\,\text{MeV}], \quad (6)$$

$$m_s/m_l = 27.2(0)(4)(0)(0) \quad [27.4(1)(4)(0)(1)], \quad (7)$$

$$m_u/m_d = 0.42(0)(1)(0)(4) \quad [0.43(0)(1)(0)(8)]. \quad (8)$$

Again, new results are on the left, and earlier ones from refs. [5,6] are in square brackets on the right. Errors are from statistics, simulation systematics, perturbation theory and electromagnetic effects. The main difference between the new and old results comes from the use of a two-loop mass renormalization constant [12], compared to the one-loop one used previously. A non-perturbative mass renormalization calculation is in progress. For further results, including Gasser-Leutwyler low-energy constants, see ref. [8].

[2] The numbers here are updated as of Lattice 2006 [8].

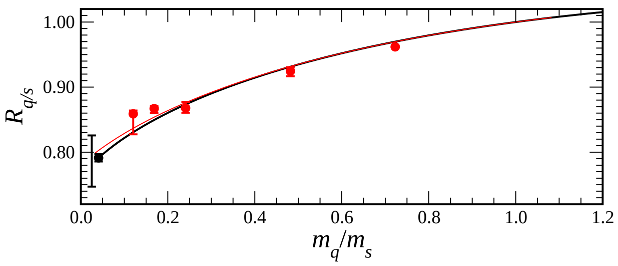

Fig. 3. (Color on-line) The ratio $R_{q/s} = f_D\sqrt{m_D}/f_{D_s}\sqrt{m_{D_s}}$ as function of m_q/m_s together with the SχPT fit. The black line and extrapolated point are obtained after removing $\mathcal{O}(a^2)$ effects from the fit, representing a continuum estimate.

3 Heavy-light meson physics

In a joint effort with the Fermilab lattice and HPQCD Collaborations, we used our full QCD ensembles to study properties of heavy-light mesons. For the heavy c and b quarks we used clover fermions with the Fermilab interpretation [13], while "asqtad" fermions where used for the light valence quarks. This allows to go much closer to the physical light-quark mass than was achieved previously. Use of SχPT, adopted to heavy-light mesons [14], makes the necessary remaining chiral extrapolation well controlled, as illustrated in fig. 3.

We found the lattice predictions [15,16]

$$f_{D_s} = 249 \pm 3 \pm 16\,\text{MeV}, \tag{9}$$
$$f_D = 201 \pm 3 \pm 17\,\text{MeV}, \tag{10}$$
$$f_{D_s}/f_D = 1.21 \pm 0.01 \pm 0.04, \tag{11}$$

compared to the later experimentally measured values from leptonic decays [17,18]

$$f_{D_s} = 283 \pm 17 \pm 7 \pm 14\,\text{MeV}, \tag{12}$$
$$f_{D^+} = 222.6 \pm 16.7^{+2.8}_{-3.4}\,\text{MeV}. \tag{13}$$

With the Fermilab lattice and HPQCD Collaborations, we are also computing form factors for semileptonic $D \to \pi/K$ and $B \to \pi/D$ decays. The heavy-to-light decay amplitudes are parametrized as

$$\langle P|V^\mu|H\rangle = f_+(q^2)(p_H + p_P - \Delta)^\mu + f_0(q^2)\Delta^\mu, \tag{14}$$

where $\Delta^\mu = (m_H^2 - m_P^2)q^\mu/q^2$. We can compare our calculation of $f_+^{(K)}(q^2)$ for $D^0 \to K^- l^+ \nu$ [19] with the recent measurement by Belle [20] (see fig. 4).

4 Conclusions

The MILC Collaboration's QCD simulations with 2 + 1 flavors have been designed to have all errors, in particular from chiral and continuum extrapolations, well controlled. For many quantities they have reached hitherto unprecedented accuracy, so that the lattice QCD calculations are starting to have an impact on current experimental physics programs. For example, using only our lattice

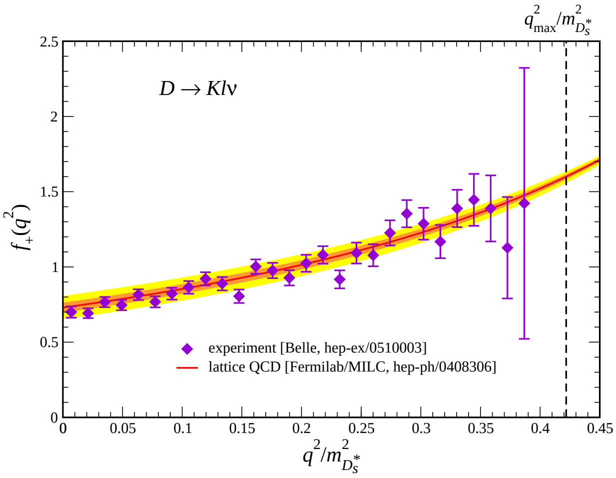

Fig. 4. (Color on-line) The form factor $f_+^{(K)}(q^2)$ for $D^0 \to K^- l^+ \nu$ compared to recent measurements by Belle [20].

calculation for f_K/f_π as a theoretical input, the CKM matrix V_{us} can be determined with an accuracy comparable to the latest PDG result. A determination of the entire CKM matrix with lattice QCD results based on the MILC ensembles of gauge configurations as the only theoretical input has been given in ref. [21].

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